Dynamics and Modeling of Turbulent Mixing in Oceanic Flows

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LONG-TERM GOALS

The long-term goal of this research is to refine and extend simple (existing) model parameterizations for turbulent diapycnal mixing for use in large scale numerical ocean models where such processes occur at the subgrid-scale. From a scientific view point, the goal is to obtain new insights into the dynamics of stratified turbulence that will translate into simple effective parameterizations for use in ocean models.

OBJECTIVES

The primary objective of this project is to bridge the gap between parameterizations and models for small-scale turbulent mixing developed from fundamental direct numerical simulations (DNS) and grid turbulence experiments (Venayagamoorthy and Stretch 2006, 2010, Stretch and Venayagamoorthy 2010) to geophysical (mesoscale and larger) scale models with an emphasis on making progress towards improved turbulent parameterizations in the ocean. The strategy is to use model-data comparisons in conjunction with theoretical modeling efforts to investigate and improve the efficacy of existing parameterizations for turbulent mixing.

APPROACH

This research takes a twin pronged approach involving process studies to assess existing turbulent parameterizations. First, through collaborations with Dr. Eric D'Asaro (University of Washington) and Dr. Louis St. Laurent (WHOI), we are looking at data possibilities that will provide high quality turbulence measurements to compare with the parameterizations developed from previous DNS/theoretical work of Dr. Venayagamoorthy (Venayagamoorthy and Stretch 2006, 2010, Stretch and Venayagamoorthy 2010). Second, we are carrying out further theoretical studies in the context of second moment closure (SMC) schemes in a RANS framework.

Turbulence closure schemes in RANS simulations such as the k-ε model (Launder and Spalding 1972) make use of a turbulent Prandtl number Pr_t to link vertical momentum and scalar fluxes. We recently derived a general relationship for the turbulent Prandtl number Pr_t for homogeneous stably stratified turbulence that is more general than previous formulations in that it is not restricted to stationary

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homogeneous flows (see Venayagamoorthy and Stretch 2010 for details). A model for Pr_t as a function of the gradient Richardson number Ri was formulated (Figure 1). In this project, we are exploring the applicability of this formulation for Pr_t in inhomogeneous stratified flows (such as boundary layer flows) in the context of a RANS framework. In line with the earlier work of Venayagamoorthy *et al.* (2003) and others (e.g. Burchard and Bolding 2001, Baumert and Peters 2004), we will be evaluating a number of turbulence closure schemes used in RANS models. In particular, the new parameterization for Pr_t developed by Venayagamoorthy and Stretch (2010) is currently being implemented in a 1-D water column model called GOTM (General Ocean Turbulence Model) developed by Burchard et al. (1999). The evaluation process will be used to refine and possibly develop new SMC parameterizations that will account for stratification effects in quantifying turbulent mixing. For example, the effect of both the buoyancy parameter ($C_{\varepsilon 3}$) and the turbulent Prandtl number in the k-ε model will be investigated by validating parameterizations against experimental and DNS/LES data sets for inhomogeneous flows such as the open channel flow configuration as well with turbulent measurements from the ocean.

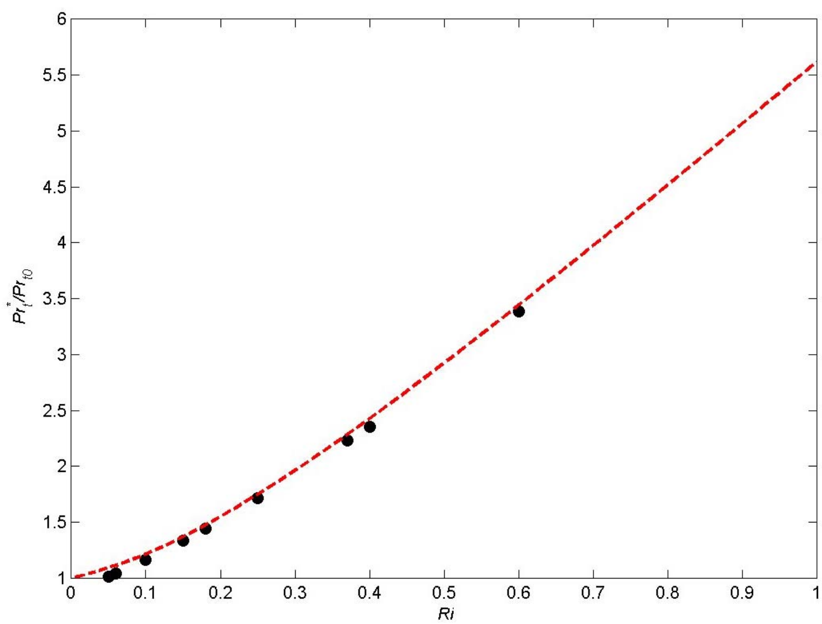

Figure 1: The turbulent Prandtl number Pr_t as a function of gradient Richardson number Ri. Black dots are DNS data (Shih et al. 2000); dashed line is the model prediction given by equation (3.6) of Venayagamoorthy and Stretch (2010).

WORK COMPLETED

Funding for this project began in the second half of FY2010 and since then we have focused our attention mainly on the theoretical modeling aspects in a RANS framework while assessing suitable data to facilitate model-data comparisons. The turbulent Prandtl number Pr_t formulation derived by Venayagamoorthy and Stretch (2010) was evaluated against three other empirically driven parameterizations for Pr_t using a simple zero-equation turbulence model for the eddy viscosity ν_t in a one-dimensional, turbulent channel flow. All four formulations for Pr_t (shown in Figure 2) were used to investigate mixing of a passive scalar plume released in both a uni-directional and oscillatory channel flow with continuous (linear) as well as two-layered density profiles (Figure 3). These test cases are somewhat artificial in that the density profile was held fixed in time similar to the simulations and experiments done by Venayagamoorthy et al. (2003) and Komori et al. (1983). However, they provide a suitable platform to elucidate the behavior of the different Pr_t schemes. Given the large range of values of gradient Richardson numbers *Ri* across the flow depth in a channel flow (for a nice theoretical discussion, see Armenio and Sarkar 2002), the mixing properties of each of the Pr_t formulations might not be immediately apparent and only through test cases like those we have performed can they be best understood.

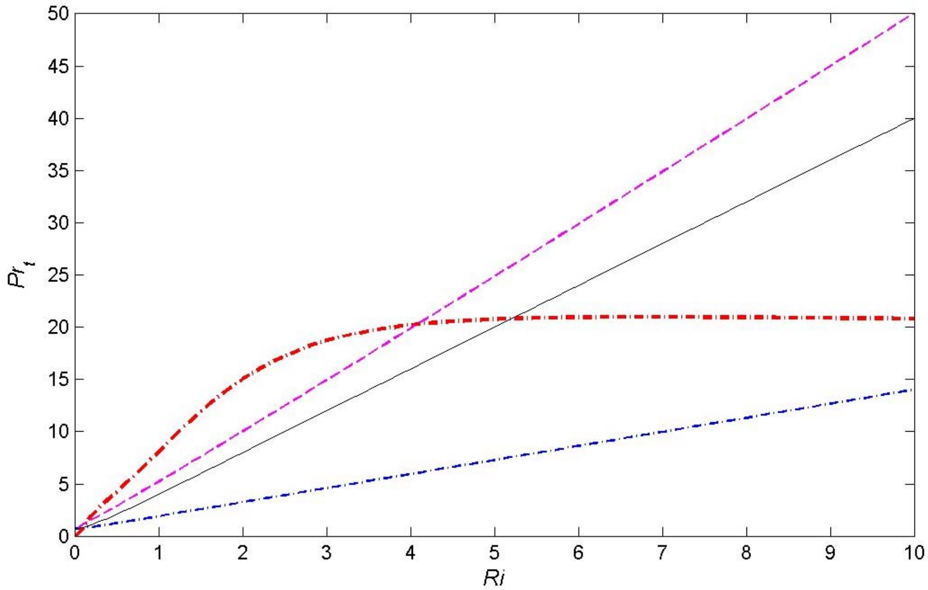

Figure 2: The turbulent Prandtl number Pr_t as function of gradient Richardson number Ri for four different models. Blue dashed-dotted line, Munk and Anderson (1948); black solid line, Venayagamoorthy and Stretch (2006); purple dashed line, Kim and Mahrt (1992); red thick dashed-dotted line, Peters et al. (1988).

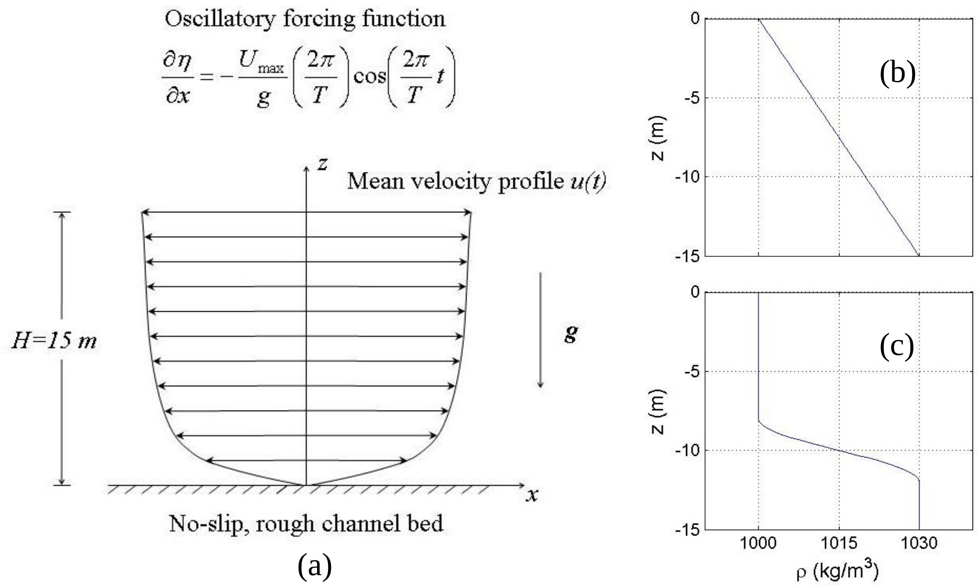

Figure 3: Schematic of the problem set up depicting (a) the tidal velocity field; (b) a continuous stratification density profile and (c) a two-layered stratification density profile (from Elliott and Venayagamoorthy 2010).

RESULTS

The effect of strong stable density stratification on the vertical mixing of a passive scalar *C* in a turbulent channel flow using four different formulations of Pr_t to describe eddy diffusivity Γ_t based on a zero-equation turbulence model for the eddy viscosity ν_t were investigated. All of the four formulations of Pr_t were tested for the two types of stratified flow conditions shown in Figures 3(b) and 3(c) for three different locations of an initial plume *C* as depicted in Figure 4(a), (b), and (c) and Figure 5(a), (b) and (c) respectively. Figure 4 shows the concentration distributions of *C* for the continuously stratified case in a tidally-driven periodic channel flow in which the velocity field changes direction as a function of time (see e.g the velocity profile shown in the schematic in Figure 3(a)). For this oscillatory flow case, the turbulence grows and decays with the phase of the tide and hence stratification effects are expected to be more dominant for this flow condition compared to the uni-directional flow cases (not shown here). Figure 5 shows the results of the mixing of *C* for the two-layered stratified tidally-driven flow.

For both of types of stratification and for all three initial release locations of the plume, the Peters et al. (1988) model predicted the quickest turbulent mixing of *C* as shown in Figure 4(d)-(f) and Figure 5(d)-(f) and the Kim and Mahrt (1992) model again predicted the slowest turbulent mixing of *C* (see Figures 4(m)-(o) and 5(m)-(o)). The final concentration profiles from the Munk and Anderson (1948) model shown in Figures 4(g)-(i) and 5(g)-(i) are not quite as well mixed as the profiles from the Peters et al. model for the oscillatory flow case. This is a direct result of the increased influence of buoyancy forces over a substantial portion of the flow depth with *Ri* values well above the cross-over value of *Ri* = 14.9 between these two models, especially for the linearly stratified case. The Venayagamoorthy and Stretch (2010) model falls in between the extremes of high and low mixing as shown in Figures 4(j)-(l) and 5(j)-(l) respectively.

For all the cases tested, the theoretical model presented by Venayagamoorthy and Stretch (2010) was the only formulation of Pr_t to predict a mixing of C that fell in between the other three models' prediction such that it never predicted the highest or lowest rate of turbulent mixing for any of the cases. The results highlight the effects of strong stratification in high Reynolds number flows and the importance of choosing an appropriate model of Pr_t in order to reasonably predict scalar mixing. It is also important to note that the formulation presented by Venayagamoorthy and Stretch (2010) is the only model considered here that is based on theoretical derivation supported by DNS data as opposed to the others which are empirically driven. Based on this premise, the VS model appears to be a balanced choice compared to the other three models discussed in this study. However, its appropriateness remains to be validated with field measurements, which is a part of this ongoing study.

We are currently extending this study to include the effects of stratification more explicitly in the formulation of the turbulent eddy viscosity in RANS turbulence models. For example, the effect of the buoyancy parameter ($C_{\varepsilon 3}$) in the two-equation k-ε model (Launder and Spalding 1972, Rodi 1993) is being explored as a function of stratification in a similar vein to the earlier work by Venayagamoorthy et al. 2003. To this end, we are continuing to seek and obtain data for strongly stratified and inhomogeneous flows to facilitate ongoing efforts to develop better turbulence models for stably stratified flows.

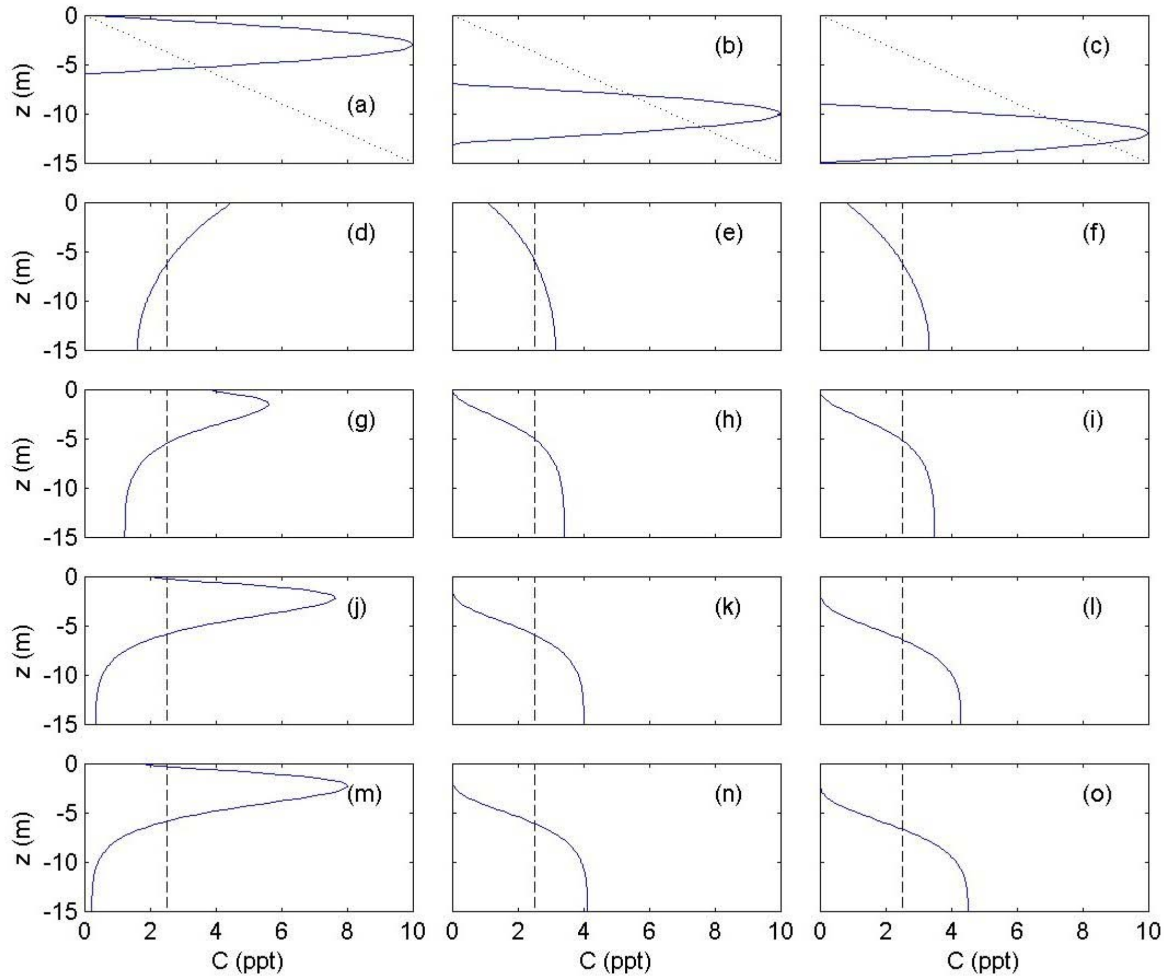

Figure 4. Concentration profiles for a passive scalar C in a uniformly stratified tidally-driven channel flow. Subplots (a) –(c) show the initial distribution of the passive scalar plume. Final concentrations profiles are shown in subplots (d) - (f) using the Peters et al. model; subplots (g) - (i) using the Munk and Anderson model; subplots (j) - (l) using the Venayagamoorthy and Stretch model; subplots (m) - (o) using the Kim and Mahrt model, respectively. Also superimposed on subplots (d) – (o) are the final profile for the unstratified case profile (from Elliott and Venayagamoorthy 2010).

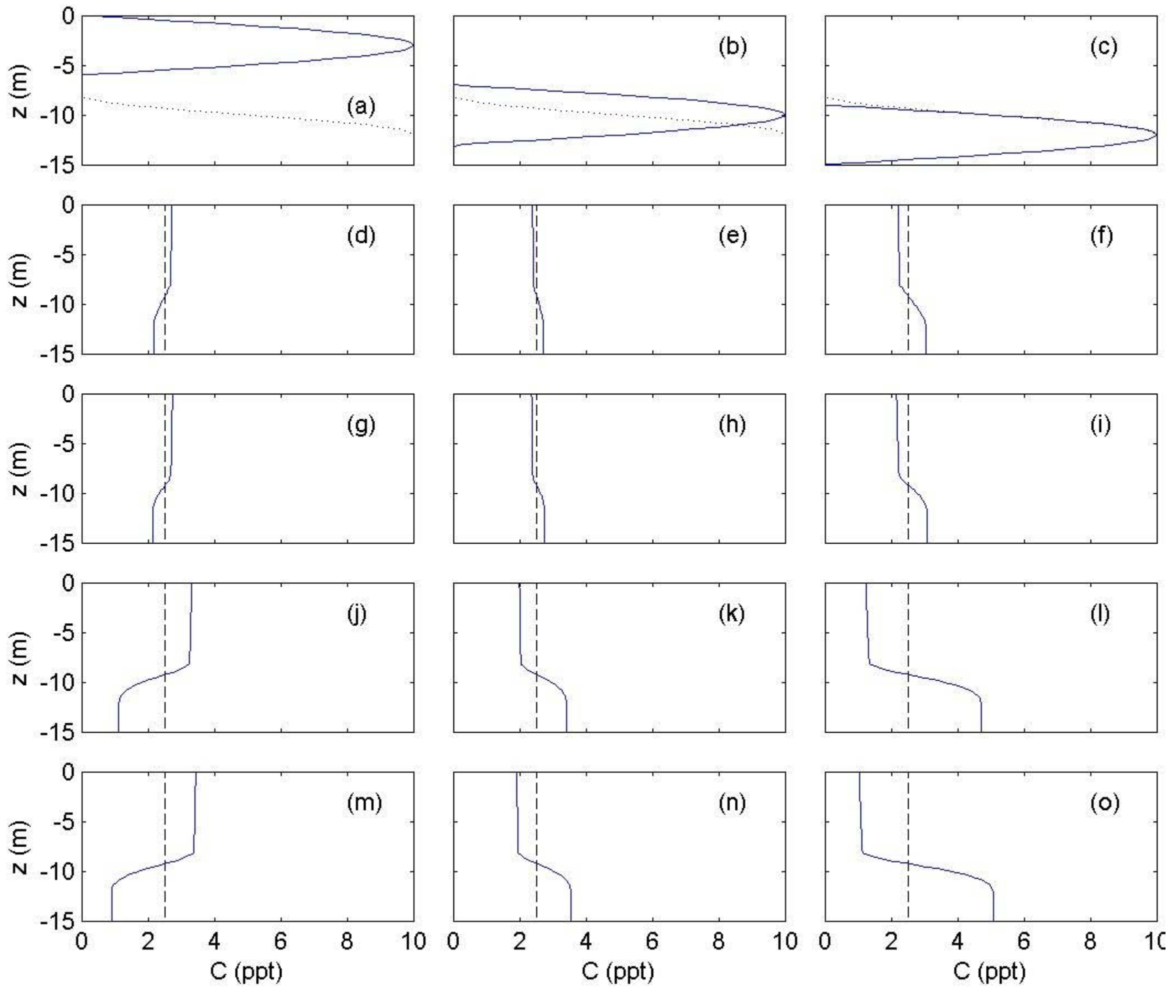

Figure 5: Concentration profiles for a passive scalar C in a two-layered stratified tidally-driven channel flow. Subplots (a) –(c) show the initial distribution of the passive scalar plume. Final concentrations profiles are shown in subplots (d) - (f) using the Peters et al. model; subplots (g) - (i) using the Munk and Anderson model; subplots (j) - (l) using the Venayagamoorthy and Stretch model; subplots (m) - (o) using the Kim and Mahrt model, respectively. Also superimposed on subplots (d) – (o) are the final profile for the unstratified case profile (from Elliott and Venayagamoorthy 2010).

IMPACT/APPLICATIONS

This project will contribute to an improved understanding of small-scale mixing processes and development of better parameterizations of such processes for applications in large-scale oceanic numerical simulations models where such processes are not explicitly resolved. In particular, this research will make contributions to the strategic ongoing efforts at ONR on assessing the effects of submesoscale ocean parameterizations and to the task of linking (small-scale) turbulence models to (larger-scale) mesoscale models.

RELATED PROJECTS

Dr. Venayagamoorthy in collaboration with Dr. Lakshmi Dasi (Colorado State University) is conducting research on the the dynamics of wall-bounded turbulent flows. The emphasis of this research (funded through start-up funds) is to understand the relevant length and time scales in wall-bounded turbulence. This ongoing effort should provide insights on how to incorporate effects of inhomogeneity into turblence models.

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PUBLICATIONS

Elliott, Z. A. and Venayagamoorthy, S. K. (2010), Evaluation of turbulent Prandtl (Schmidt) number parameterizations for stably stratified environmental flows, *Dyn. Atmos. Oceans* (submitted).

HONORS/AWARDS/PRIZES

Subhas Karan Venayagamoorthy, Lorenz G. Straub Award for the most meritorious PhD dissertation in hydraulic engineering and fluid mechanics, St. Anthony Falls Laboratory, University of Minnesota, 2010.

Subhas Karan Venayagamoorthy, Outstanding Faculty Performance Award, Department of Civil and Environmental Engineering, Colorado State University, 2010.

Subhas Karan Venayagamoorthy, Selected as the Borland Chair of Hydraulics, Department of Civil and Environmental Engineering, Colorado State University, 2010.